## Study Questions for Midterm 2, Data Structures \& Algorithms

You are encouraged to discuss questions and solutions with your classmates or others. I have no solutions to provide but can answer questions you may have.

## 1. Heaps

Given the binary min-heap below:
a) Show the heap after inserting an element with the value of 7 and then the value of 2.
b) Show the heap after extracting the min value (use the original heap, not after the inserts)


| 1 | 3 | 6 | 5 | 9 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 2 | 3 | 4 | 5 |

c) If we are using the array representation for the heap as shown above, what would be problematic about representing a tree like the one below?

d) A d-ary heap is like a binary heap, but (with one possible exception) non-leaf nodes have $d$ children instead of 2 children.
a. Describe how would you represent a d-ary heap in an array in terms of organizing parents and children.
b. What is the height of a d-ary heap of $n$ elements in terms of $n$ and $d$ ?
e) Describe an efficient algorithm to modify the key value in a heap. That is, given a valid heap, you want to modify a key in the heap to be either smaller or larger, and fix the heap so it is a valid heap again.

## 2. Tries

a. Suppose you want to store a dictionary of words. What are the pros and cons of using a trie vs. using a sorted list with binary search to look up a word? Be specific in terms of the runtime and space requirements.
b. Consider a trie that holds a dictionary of words, like we discussed in the notes. Give pseudocode for an algorithm that given a prefix, prints out all words that start with that prefix.

## 3. Treaps

Consider the following treap, where the red is the BST value and the blue is the heap priority:

a. Show the rotations to insert a new node with $k e y=18$, priority=79
b. Show the rotations to delete the node with key $=11$ on the original treap

## 4. Short Answer

a. What are the minimum and maximum number of elements in a heap of height $h$ ?
b. Where in a max-heap might the smallest element reside?
c. Under what conditions would it be more efficient to use a linked list to store all of the child nodes for a trie rather than store the child nodes in an array?
d. You have an unsorted array of n elements. You want to find the k smallest elements, where k is much smaller than n . Describe an efficient way to find these k elements.

## 5. Sorting

Below are the contents of an array as two algorithms are sorting it. Each line does not necessarily represent the next step in the algorithm, but only some later step during the course of the algorithm's execution. Key elements are depicted in bold. Identify each algorithm.

Algorithm 1

| 12 | 39 | 2 | 94 | 23 | 77 | 52 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 12 | $\mathbf{3 9}$ | 2 | 94 | 23 | 77 | 52 | 9 |
| $\mathbf{2}$ | 12 | 39 | 94 | 23 | 77 | 52 | 9 |
| 2 | 12 | 39 | $\mathbf{9 4}$ | 23 | 77 | 52 | 9 |
| 2 | 12 | $\mathbf{2 3}$ | 39 | 94 | 77 | 52 | 9 |
| 2 | 12 | 23 | 39 | $\mathbf{7 7}$ | 94 | 52 | 9 |
| 2 | 12 | 23 | 39 | $\mathbf{5 2}$ | 77 | 94 | 9 |
| 2 | $\mathbf{9}$ | 12 | 23 | 39 | 52 | 77 | 94 |

Algorithm 2

| 12 | 39 | 2 | 94 | 23 | 77 | 52 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{9 4}$ | $\mathbf{3 9}$ | $\mathbf{7 7}$ | $\mathbf{1 2}$ | $\mathbf{2 3}$ | $\mathbf{2}$ | $\mathbf{5 2}$ | $\mathbf{9}$ |
| 77 | 39 | 52 | 12 | 23 | 2 | 9 | $\mathbf{9 4}$ |
| 52 | 39 | 9 | 12 | 23 | 2 | $\mathbf{7 7}$ | 94 |
| 39 | 23 | 9 | 12 | 2 | $\mathbf{5 2}$ | 77 | 94 |
| 23 | 12 | 9 | 2 | $\mathbf{3 9}$ | 52 | 77 | 94 |
| 12 | 2 | 9 | $\mathbf{2 3}$ | 39 | 52 | 77 | 94 |
| 9 | 2 | $\mathbf{1 2}$ | 23 | 39 | 52 | 77 | 94 |
| 2 | $\mathbf{9}$ | 12 | 23 | 39 | 52 | 77 | 94 |

