Functional Programming in Scheme

CSCE A331

Functional Programming

- Online textbook: http://www.htdp.org/
- Original functional language is LISP
  - LISt Processing
  - The list is the fundamental data structure
  - Developed by John McCarthy in the 60's
    - Used for symbolic data processing
    - Example apps: symbolic calculations in integral and differential calculus, circuit design, logic, game playing, AI
    - As we will see the syntax for the language is extremely simple
  - Scheme
    - Descendant of LISP

Functional Languages

- "Pure" functional language
  - Computation viewed as a mathematical function mapping inputs to outputs
  - No notion of state, so no need for assignment statements (side effects)
  - Iteration accomplished through recursion
- In practicality
  - LISP, Scheme, other functional languages also support iteration, assignment, etc.
  - We will cover some of these "impure" elements but emphasize the functional portion
- Equivalence
  - Functional languages equivalent to imperative
    - Core subset of C can be implemented fairly straightforwardly in Scheme
    - Scheme itself implemented in C
    - Church-Turing Thesis

Lambda Calculus

- Foundation of functional programming
- Developed by Alonzo Church, 1941
- A lambda expression defines
  - Function parameters
  - Body
- Does NOT define a name; lambda is the nameless function. Below x defines a parameter for the unnamed function:

$$(\lambda x \cdot x * x)$$

Lambda Calculus

- Given a lambda expression

$$(\lambda x \cdot x * x)$$

- Application of lambda expression

$$((\lambda x \cdot x * x)2) \rightarrow 4$$

- Identity $(\lambda x \cdot x)$

- Constant 2: $(\lambda x \cdot 2)$

Lambda Calculus

- Any identifier is a lambda expression
- If M and N are lambda expressions, then the application of *M* to *N*, *(MN)* is a lambda expression
- An abstraction, written $(\lambda x \cdot M)$ where *x* is an identifier and M is a lambda expression, is also a lambda expression

Lambda Calculus

$$LambdaExpression \rightarrow ident \mid (MN) \mid (\lambda \ \ ident \cdot M)$$

$$M \rightarrow LambdaExpression$$

$$N \rightarrow LambdaExpression$$

Examples

$$x$$

$$(\lambda x \cdot x)$$

$$((\lambda x \cdot x)(\lambda y \cdot y))$$

Lambda Calculus

First Class Citizens

- Functions are *first class citizens*
  - Can be returned as a value
  - Can be passed as an argument
  - Can be put into a data structure as a value
  - Can be the value of an expression

$$((\lambda x \cdot x * x)(\lambda y \cdot 2)) = (\lambda x \cdot 2 * 2) = 4$$

$$((\lambda x \cdot (\lambda y \cdot x+y))\ 2\ 1) = ((\lambda y \cdot 2+y)\ 1) = 3$$

Lambda Calculus

Functional programming is essentially an applied lambda calculus with built in

- constant values
- functions

E.g. in Scheme, we have (* x x) for x*x instead of $\lambda x \cdot x * x$

Functional Languages

- Two ways to evaluate expressions
- Eager Evaluation or Call by Value
  - Evaluate all expressions ahead of time
  - Irrespective of if it is needed or not
  - May cause some runtime errors

- Example

(foo 1 (/ 1 x)) Problem; divide by 0

Lambda Calculus

- Lazy Evaluation
  - Evaluate all expressions only if needed
    (foo 1 (/ 1 x))   ; (/ 1 x) not needed, so never eval'd
  - Some evaluations may be duplicated
  - Equivalent to call-by-name
  - Allows some types of computations not possible in eager evaluation
- Example
  - Infinite lists
    - E.g,. Infinite stream of 1's, integers, even numbers, etc.
  - Replaces tail recursion with lazy evaluation call
  - Possible in Scheme using (force/delay)

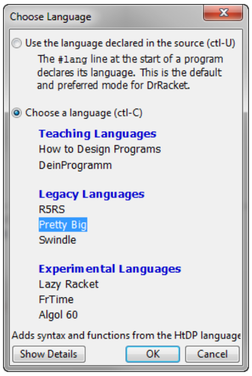
Running Scheme for Class

- A version of Scheme called Racket (formerly PLT/Dr Scheme) is available on the Windows machines in the NSB/ENGR Labs
- Download: http://racket-lang.org/
- Unix, Mac versions also available if desired

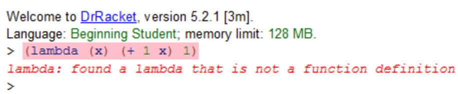
Racket

- You can type code directly into the interpreter and Scheme will return with the results:

Make sure right Language is selected



Use the **"Pretty Big"** language choice – it is closer to Scheme than others



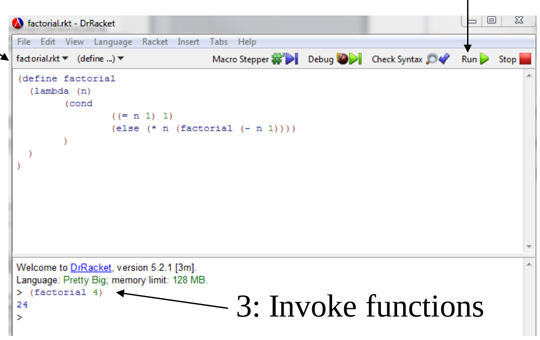
Racket – Loading Code

- You can open code saved in a file. Racket uses the extension ".rkt" so consider the following file "factorial.rkt" created with a text editor or saved from Racket:

```
(define factorial
  (lambda (n)
     (cond
          ((= n 1) 1)
          (else (* n (factorial (- n 1))))
     )
  )
)
```

1: Open

2: Run



3: Invoke functions

Functional Programming Overview

- Pure functional programming
  - No implicit notion of state
  - No need for assignment statement
    - No side effect
  - Looping
    - No state variable
    - Use Recursion
- Most functional programming languages have side effects, including Scheme
  - Assignments
  - Input/Output

Scheme Programming Overview

- Refreshingly simple
  - Syntax is learned in about 10 seconds
- Surprisingly powerful
  - Recursion
  - Functions as first class objects (can be value of an expression, passed as an argument, put in a data structure)
- Implicit storage management (garbage collection)
- Lexical scoping
  - Earlier LISPs did not do that (dynamic)
- Interpreter
  - Compiled versions available too

Expressions

- Syntax - Cambridge Prefix
  - Parenthesized
  - (* 3 4)
  - (* (+ 2 3) 5)
  - (f 3 4)
- In general:
  - (functionName arg1 arg2 …)
- Everything is an expression
  - Sometimes called s-expr (symbolic expr)

Expression Evaluation

- Replace symbols with their bindings
- Constants evaluate to themselves
  - 2, 44, #f
  - No nil in Racket; use ‘()
    - Nil = empty list, but Racket does have **empty**
- Lists are evaluated as function calls written in Cambridge Prefix notation

  (+ 2 3)

  (* (+ 2 3) 5)

Scheme Basics

- **Atom**
  - Anything that can’t be decomposed further
    - a string of characters beginning with a letter, number or special character other than (or)
    - e.g. 2, #t, #f, “hello”, foo, bar
    - #t = true
    - #f = false
- **List**
  - A list of atoms or expressions enclosed in ()
  - (), empty,(1 2 3), (x (2 3)), (()()())

Scheme Basics

- **S-expressions**
  - Atom or list

- () or empty
  - Both atom and a list

- Length of a list
  - Number at the top level

Quote

- If we want to represent the literal list (a b c)
  – Scheme will interpret this as apply the arguments b and c to function a
- To represent the literal list use “quote”
  – (quote x) → x
  – (quote (a b c)) → (a b c)
- Shorthand: single quotation mark

  ‘a == (quote a)

  ‘(a b c) == (quote (a b c))

Global Definitions

- Use define function

```
(define f 20)
(define evens ‘(0 2 4 6 8))
(define odds ‘(1 3 5 7 9))
(define color ‘red)
(define color blue)          ; Error, blue undefined
(define num f)               ; num = 20
(define num ‘f)              ; symbol f
(define s  “hello world”)    ; String
```

Lambda functions

- Anonymous functions
  - (lambda (<formals>) <expression>)
  - (lambda (x) (* x x))
  - ((lambda (x) (* x x)) 5) → 25
- Motivation
  - Can create functions as needed
  - Temporary functions : don't have to have names
- Can not use recursion

Named Functions

- Use define to bind a name to a lambda expression

```
(define square (lambda (x) (* x x)))
(square 5)
```

- Using lambda all the time gets tedious; alternate syntax:

(define (<function name> <formals>) <expression1> <expression2> …)

Last expression evaluated is the one returned

```
(define (square x) (* x x))
(square 5) → 25
```

Conditionals

(if <predicate> <expression1> <expresion2>)
- Return value is either expr1 or expr2

(cond (P1 E1)
(P2 E2)
(P_n E_n)
(else E_{n+1}))

- Returns whichever expression is evaluated

Common Predicates

- Names of predicates end with ?
  - Number? : checks if the argument is a number
  - Symbol? : checks if the argument is a symbol
  - Equal? : checks if the arguments are structurally equal
  - Null? : checks if the argument is empty
  - Atom? : checks if the argument is an atom
    - Appears undefined in Racket but can define ourselves
  - List?   : checks if the argument is a list

Conditional Examples

- (if (equal? 1 2) ‘x ‘y) ; y
- (if (equal? 2 2) ‘x ‘y) ; x
- (if (null? ‘()) 1 2) ; 1
- (cond

```
    ((equal? 1 2)   1)
    ((equal? 2 3)   2)
    (else 3))                    ;   3
```

- (cond

```
    ((number?  ‘x)    1)
    ((null?  ‘x) 2)
    ((list?   ‘(a b c))  (+ 2 3))      ;  5
 )
```

Dissecting a List

- **Car** : returns the first argument
  - (car ‘(2 3 4))
  - (car ‘((2) 4 4))
  - Defined only for non-null lists
- **Cdr** : (pronounced “could-er”) returns the rest of the list
  - Racket: list must have at least one element
  - Always returns a list
    - (cdr ‘(2 3 4))
    - (cdr ‘(3))
    - (cdr ‘(((3))))
- Compose
  - (car (cdr ‘(4 5 5)))
  - (cdr (car ‘((3 4))))

Shorthand

- (cadr x) = (car (cdr x))
- (cdar x) = (cdr (car x))
- (caar x) = (car (car x))
- (cddr x) = (cdr (cdr x))
- (cadar x) = (car (cdr (car x)))
- … etc… up to 4 levels deep in Racket
- (cddadr x) = ?

Why Car and Cdr?

- Leftover notation from original implementation of Lisp on an IBM 704
- CAR = Contents of Address part of Register
  - Pointed to the first thing in the current list
- CDR = Contents of Decrement part of Register
  - Pointed to the rest of the list

Building a list

- **Cons**
  - Cons(truct) a new list from first and rest
  - Takes two arguments
  - Second should be a list
    - If it is not, the result is a “dotted pair” which is typically considered a malformed list
  - First may or may not be a list
  - Result is always a list

Building a list

X = 2 and Y = (3 4 5) : (cons x y) → (2 3 4 5)

X = () and Y =(a b c) : (cons x y) → (() a b c)

X = a and Y =() : (cons x y) → (a)

- What is
  - (cons 'a (cons 'b (cons 'c '())))
  - (cons (cons ‘a (cons ‘b ‘())) (cons ‘c ‘()))

Numbers

- Regular arithmetic operators are available
  - +, -, *, /
  - May take variable arguments
  - (+ 2 3 4), (* 4 5 9 11)
- (/ 9 2) → 4.5 ; (quotient 9 2) → 4
- Regular comparison operators are available
  - < > <= >= =
    - E.g. (= 5 (+ 3 2)) → #t

  = only works on numbers, otherwise use equal?

Example

- Sum all numbers in a list

```
(define (sumall list)
  (cond
   ((null? list) 0)
   (else (+ (car list) (sumall (cdr list))))))
```

Sample invocation: (sumall ‘(3 45 1))

Example

- Make a list of n identical values

```
(define (makelist n value)
   (cond
     ((= n 0) '())
     (else
       (cons value (makelist (- n 1) value))
     )
   )
  )
```

In longer programs, careful matching parenthesis.

Example

- Determining if an item is a member of a list

```
(define (member? item list)
   (cond ((null? list) #f)
         ((equal? (car list) item) #t)
         (else (member? item (cdr list)))
   )
  )
```

Scheme already has a built-in (member item list) function
that returns the list after a match is found

Example

- Remove duplicates from a list

```
(define (remove-duplicates list)
   (cond ((null? list) '())
        ((member? (car list) (cdr list))
         (remove-duplicates (cdr list)))
        (else
         (cons (car list) (remove-duplicates (cdr list))))
   )
 )
```