Sample Midterm Exam **Name: _____**
CS351
100 Points Total

Please write neatly and show your work. The exam is open book, open notes, and you may use any computing devices. You have one hour and 15 minutes. Good luck!

1. Short Answer (12 pts). Provide brief 1-3 sentence answers for the following questions.

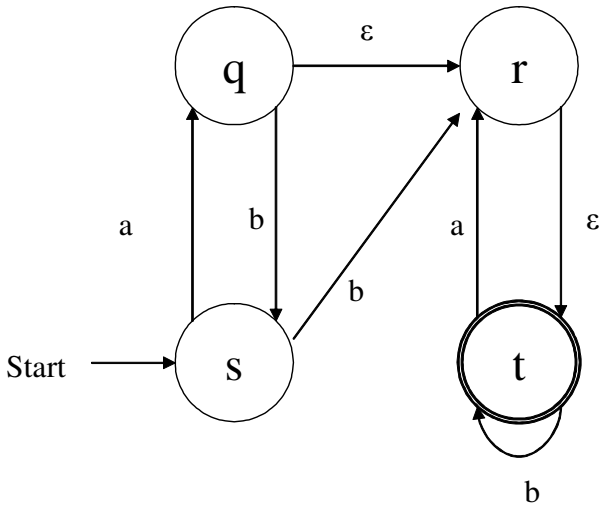
a. Argue that the set of decidable languages (i.e. languages that are decidable by a Turing Machine) is closed under complement.

b. With the pumping lemma for context-free languages, we stated that there exists a string s = uvwxy where one of the conditions is that $|vx| \neq \varepsilon$. Give an argument why we can claim that $|vx| \neq \varepsilon$.

c. How could you prove that a regular expression and an ε-NFA both represent the same language?

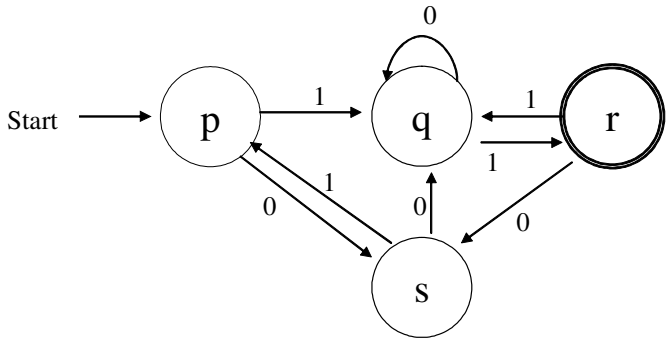
2. **DFA's (12 pts).** Design a DFA that recognizes the substring "aab" from a larger string over $\sum = \{a,b\}$. Rather than stop when a match is found, your DFA should find all occurrences of the substring in the larger string.

3. **NFA's. (12 pts).** Convert the following ε-NFA into a DFA.



4. DFA State Elimination (12 pts).

Convert the DFA below to a regular expression using the state elimination technique. Be sure to show your work in the elimination of states, do not jump straight to a regular expression.



5. **Regular Expressions. (9 pts).** Generate regular expressions for the following:

   a. The set of strings of 0's and 1's that has no two adjacent 1's.
   b. The set of strings of 0's and 1's that has no two adjacent 0's.
   c. The set of strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

6. **Context-Free Languages (12 pts).** Give a context free grammar that generates the same language as the regular expression **0*(1+ε)(0+1)***

7. **Pumping Lemma (16 pts).**

   Use the pumping lemma to show the following language is not regular:
   $\{ 0^n \mid n$ is a perfect cube; i.e. 1, 8, 27, 64, etc.$\}$

8. Turing Machines (15 pts).

a) Describe, either formally or informally (i.e. in English), a Turing Machine that recognizes the language:

$L = \{ a^n b^n \mid n \geq 1 \}$

where $\Sigma = \{ a,b \}$. You can design the tape alphabet as needed.

b) Let A be the language containing only the single string s, where:

s = 0 if God exists
s = 1 if God does not exist

Is A decidable? Explain why or why not. The answer has no relation to your religious beliefs, but for purposes of this problem assume that the question of whether or not God exists has an unambiguous YES or NO answer.

c) Argue that A = {<M> | M is a DFA that doesn't accept any string containing an odd number of 1's } is decidable.